Due: Tuesday, 2/26/2008 before class

1. Any two minimal DFAs recognizing the same language are isomorphic (identical up to renaming the states). Must this also be true of NFAs? Prove this or give a counterexample.

2. Give context-free grammars for
   (a) the complement of $\{(a^n b)^n \mid n \geqslant 1\}$ and
   (b) the language of non-palindromes $\{x \in \{a, b\}^* \mid x \neq x^R\}$.

3. Suppose we modify the definition of a PDA so that instead of requiring $\delta(q, a, A)$ to be a finite set $\{(q_1, \gamma_1), \ldots, (q_k, \gamma_k)\}$ representing the nondeterministic choices of the PDA, we allow the PDA to nondeterministically choose among a potentially infinite set, but this set must be context-free. More formally, we allow

   $$\delta(q, a, A) = (q_1 \times L_1) \cup (q_2 \times L_2) \cup \cdots \cup (q_k \times L_k),$$

   where each L_i is a context-free language. We accept by empty stack.

   Prove or disprove: the class of languages accepted by these more powerful PDAs is precisely the class of context-free languages.

4. Show that $C = \{w \in \{1, 2, 3, 4\}^* \mid n_1(w) = n_2(w) \text{ and } n_3(w) = n_4(w)\}$ is not context-free. (For a string w and letter a, $n_a(w)$ denotes the number of times a occurs in w.)

5. Let G be a CFG in Chomsky normal form with k symbols. Show that if G generates some string with a derivation having at least 2^k steps, then $L(G)$ is infinite. Prove all claims that you make.

6. **(Extra-credit)**
   If L is a language, define $c(L) = \{w \mid w \text{ can be written } w = xy, \text{ so that } yx \in L\}$. Show that
   (a) If L is regular, then so is $c(L)$.
   (b) If L is context-free, then so is $c(L)$.