Homework 1 CSE 555 Fall 2007 Arizona State University

Due: Thursday, 2/7/2008 before class

1. Let $\Sigma = \{0, 1\}$. You are given two strings $u, v \in \Sigma^*$ of nonequal length (say $n = |u| > |v| = m$). Explain how to construct a DFA $M(u, v)$ that distinguishes u from v (that is, a DFA that accepts exactly one of the two strings, and rejects the other). Show how to find a DFA $M(u, v)$ that uses only $O(\log n)$ states.

2. Let $F = \{a^i b^j c^k \mid \text{if } i = 1 \text{ then } j = k\}$. Show that F behaves like a regular language in the pumping lemma (that is, show that the pumping lemma doesn't work on F). Then show that F is not regular.

3. Let x and y be strings and L be a language. We say that x and y are *distinguishable by* L if there exists a string z such that exactly one of xz and yz belongs to L; otherwise, for every string z, the strings xz and yz either both belong to L or both do not belong to L, and we say that x and y are *indistinguishable by* L. If x and y are indistinguishble by L, we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

4. Let L be a language and X a set of strings. Say that X is *pairwise distinguishable by* L, if every two distinct strings in X are distinguishable by L. Define the *index of* L to be the maximum number of elements in any set that is pairwise distinguishable by L. The index of L may be finite or infinite.

   (a) Show that if L is recognized by a DFA with k states, L has index at most k.

   (b) Show that if the index of L is a finite number k, it is recognized by a DFA with k states.

   (c) Conclude that L is regular if and only if it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.