

Due: Thursday, 1/31/2008 before class

1. Let  $A, B \subseteq \Sigma^*$ . Define  $A/B = \{x \in \Sigma^* \mid \exists y \in B : xy \in A\}$ . For example, if  $L_1 = a^+bc^+$ ,  $L_2 = bc^+$  and  $L_3 = c^+$ . Then  $L_1/L_2 = a^+$  and  $L_1/L_3 = a^+bc^*$ .
  - (a) Suppose  $A$  is regular. Prove that  $A/B$  is also regular. (Do not make any assumptions on  $B$ .)
  - (b) Suppose now both  $A$  and  $B$  are regular and you are given DFAs  $M_A$  and  $M_B$  such that  $A = L(M_A)$  and  $B = L(M_B)$ . Show how to construct a DFA for  $A/B$ .
2.
  - (a) Find a nonregular language  $A$  such that  $AA^R$  is regular.
  - (b) Find a nonregular language  $A$  such that  $A^*$  is regular.
  - (c) Let  $A$  be an arbitrary language over the one-letter alphabet  $\Sigma = \{a\}$ . Prove that  $A^*$  is regular.
3. In an *extended regular expression*, intersection and complement can be used in addition to the three regular operations. Show how to write  $(aba)^*$  as a star-free extended regular expression (that is, you are allowed to use intersection, union, concatenation and complementation, but not the Kleene star operations  $^*$  or  $^+$ ).