Due: Thursday 9/27 before 10:30

1. You're given a graph $G = (V, E)$ and an integer $f$. Describe a polynomial-time algorithm that finds a largest subset $F$ of edges in $E$ such that for every $v \in V$, at most $f$ edges of $F$ are incident on $v$. You may use a maximum matching algorithm as a subroutine.

2. You're given a graph $G = (V, E)$ and for every $v \in V$, an integer $f_v$. Describe a polynomial-time algorithm that finds a largest subset $F$ of edges in $E$ such that for every $v \in V$, at most $f_v$ edges of $F$ are incident on $v$. You may use a maximum matching algorithm as a subroutine.

3. You're given a graph $G = (V, E)$ and for every $v \in V$, two integers, $\ell_v$ and $u_v$. Describe a polynomial-time algorithm that finds a largest subset $F$ of edges in $E$ such that for every $v \in V$, the number $x_v$ of edges in $F$ incident on $v$ satisfies $\ell_v \leq x_v \leq u_v$.

4. Show that in a bipartite graph, the cardinality of the maximum matching equals the cardinality of the smallest set of vertices that covers all edges (that is, every edge is incident on a vertex of this set). How about the nonbipartite case?

5. You're given a graph $G = (V, E)$, a partition of $V$ into $A$ and $B$ (that is, $A \cup B = V$, $A \cap B = \emptyset$), and two integers $a$ and $b$. Give a polynomial time algorithm that finds a matching $M$ in $G$ such that at least $a$ vertices of $A$ and at most $b$ vertices of $B$ are covered by the matching.