

Due: Thursday 9/13 before 10:30

1. What is the maximum number of different values for s - t cuts in a directed graph on n vertices?
2. Prove that every weighted undirected graph has a flow-equivalent tree which is a path.
3. Let $A, B \subseteq V$ be two subsets of the vertex-set of the (undirected) graph $G = (V, E)$. Let $c : E \rightarrow \mathbf{R}_+$ be a cost/weight/capacity function defined on the set of edges of G .
 - (a) Show that
$$c(\delta(A)) + c(\delta(B)) \geq c(\delta(A \cap B)) + c(\delta(A \cup B)).$$
 - (b) Show that if both $\delta(A)$ and $\delta(B)$ are minimum s - t cuts in G , then so are $\delta(A \cap B)$ and $\delta(A \cup B)$.
4. Prove that in a matrix, the maximum number of non-zero entries, no two in the same line (row or column), is equal to the minimum number of lines that include all non-zero entries.